# Pricing under the Generalized Markov Chain Choice Model: Learning through Large-Scale Click Behaviors

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Service Science Student Paper Competition

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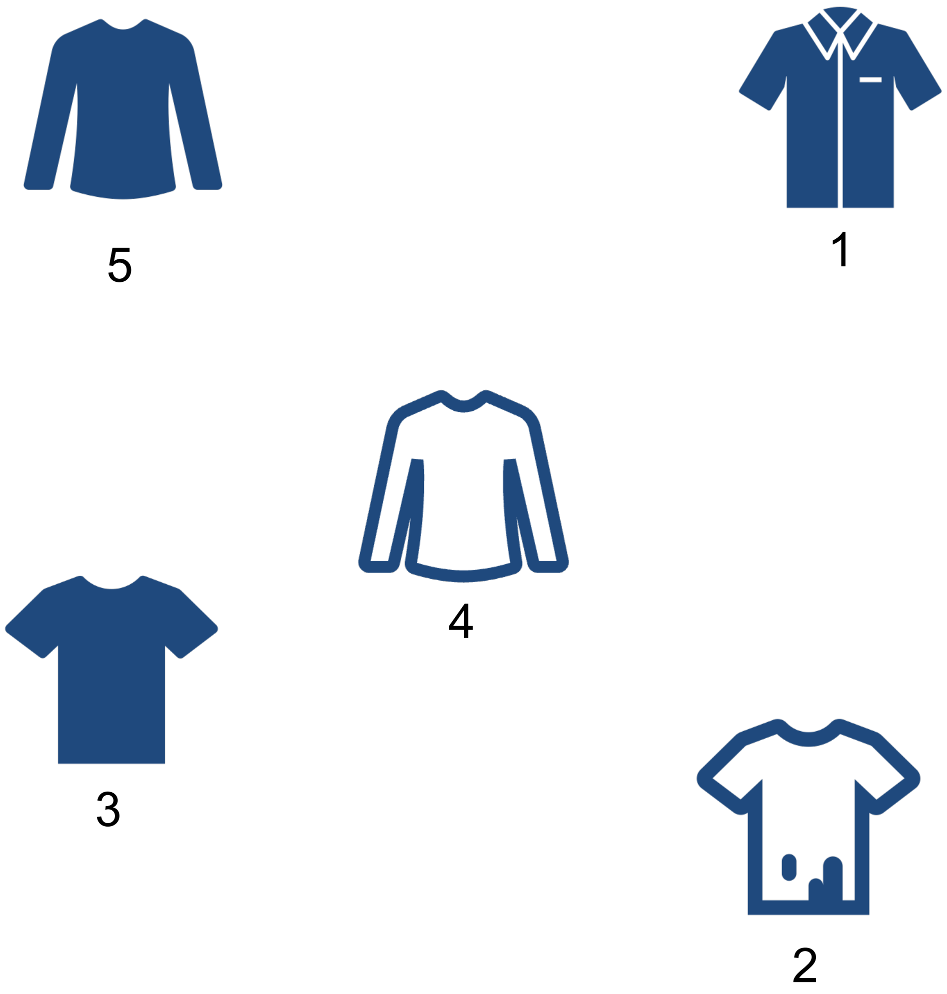
# Example: click behaviors before buying a shirt on a website



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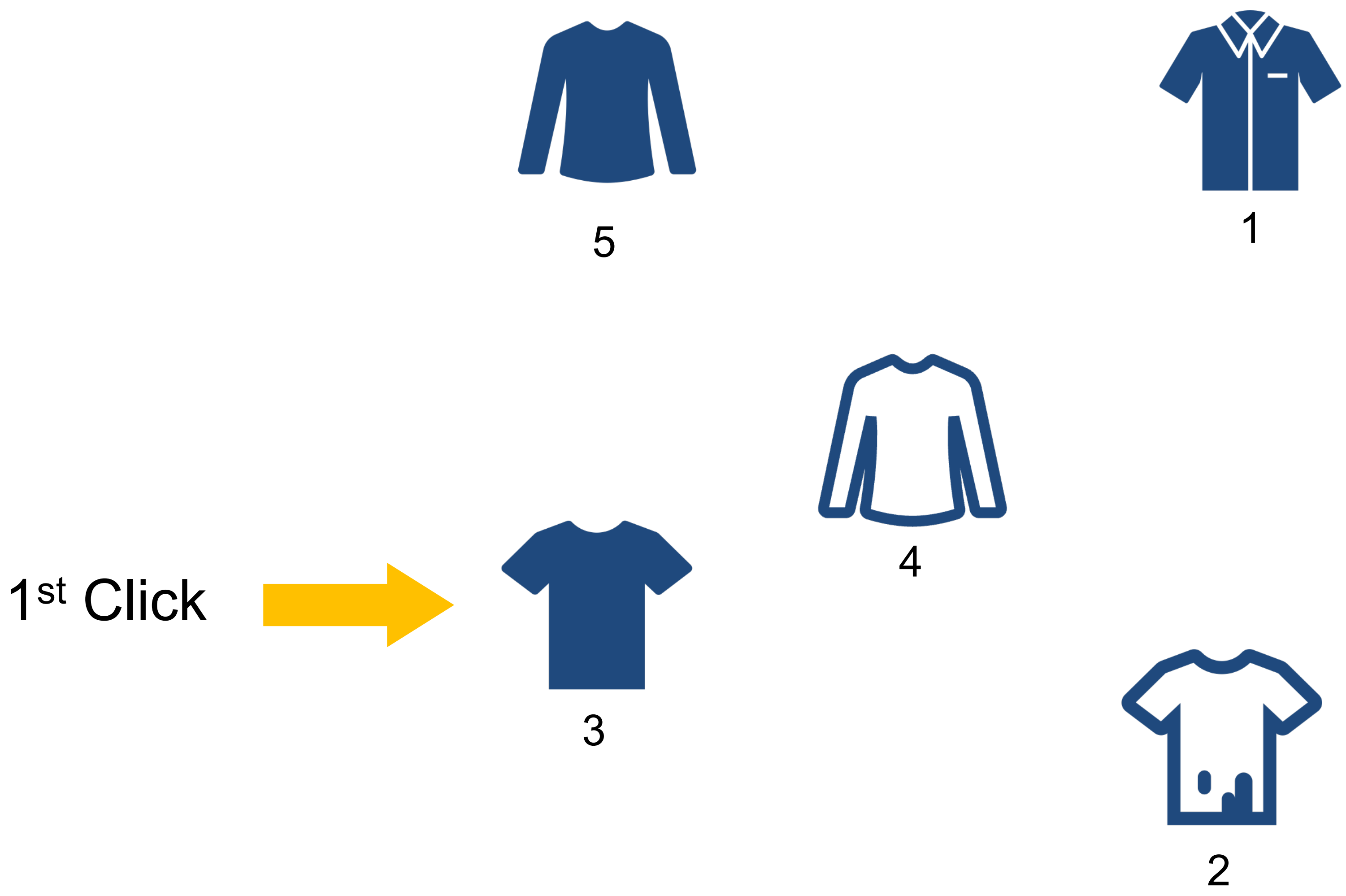

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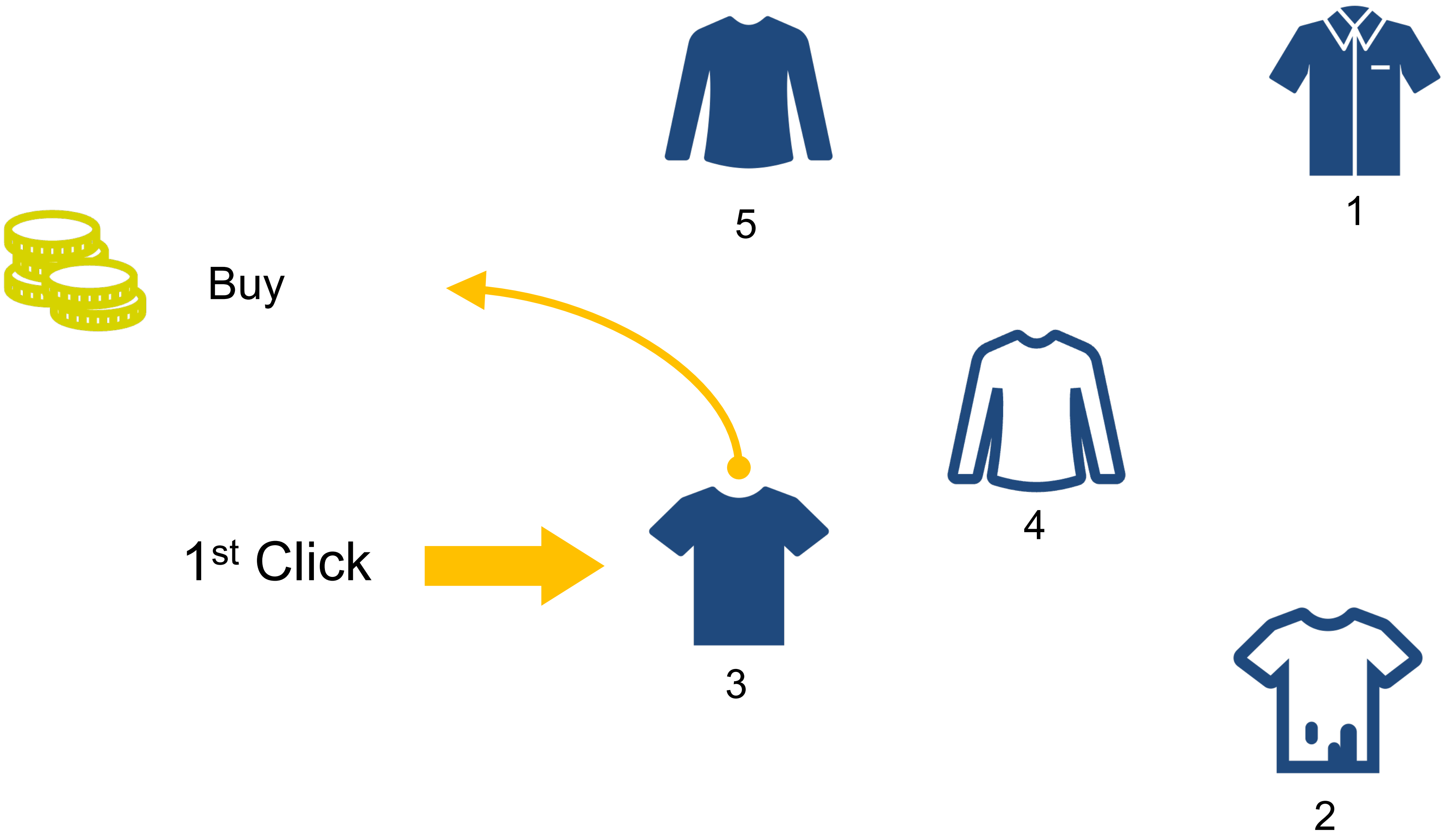

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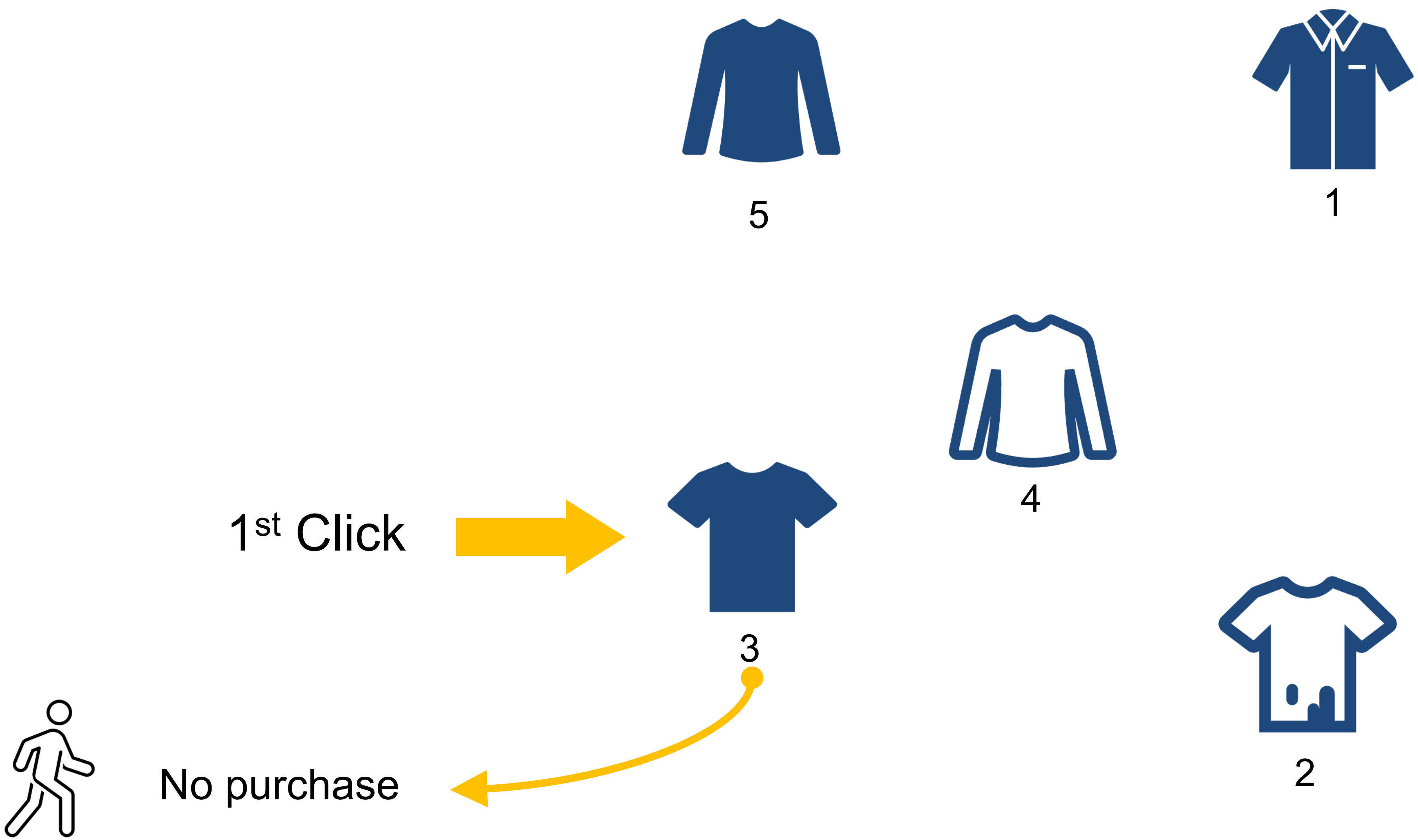

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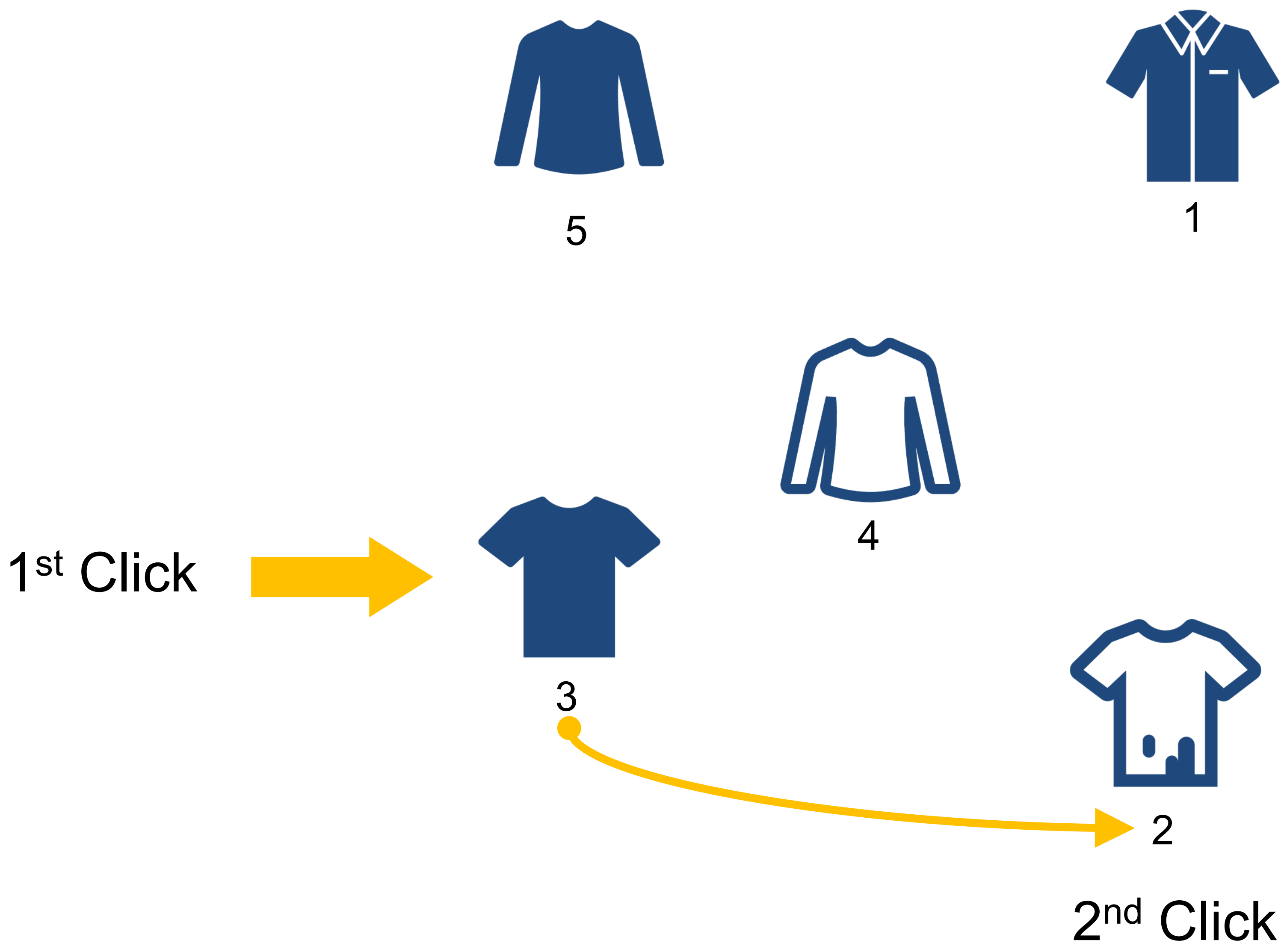

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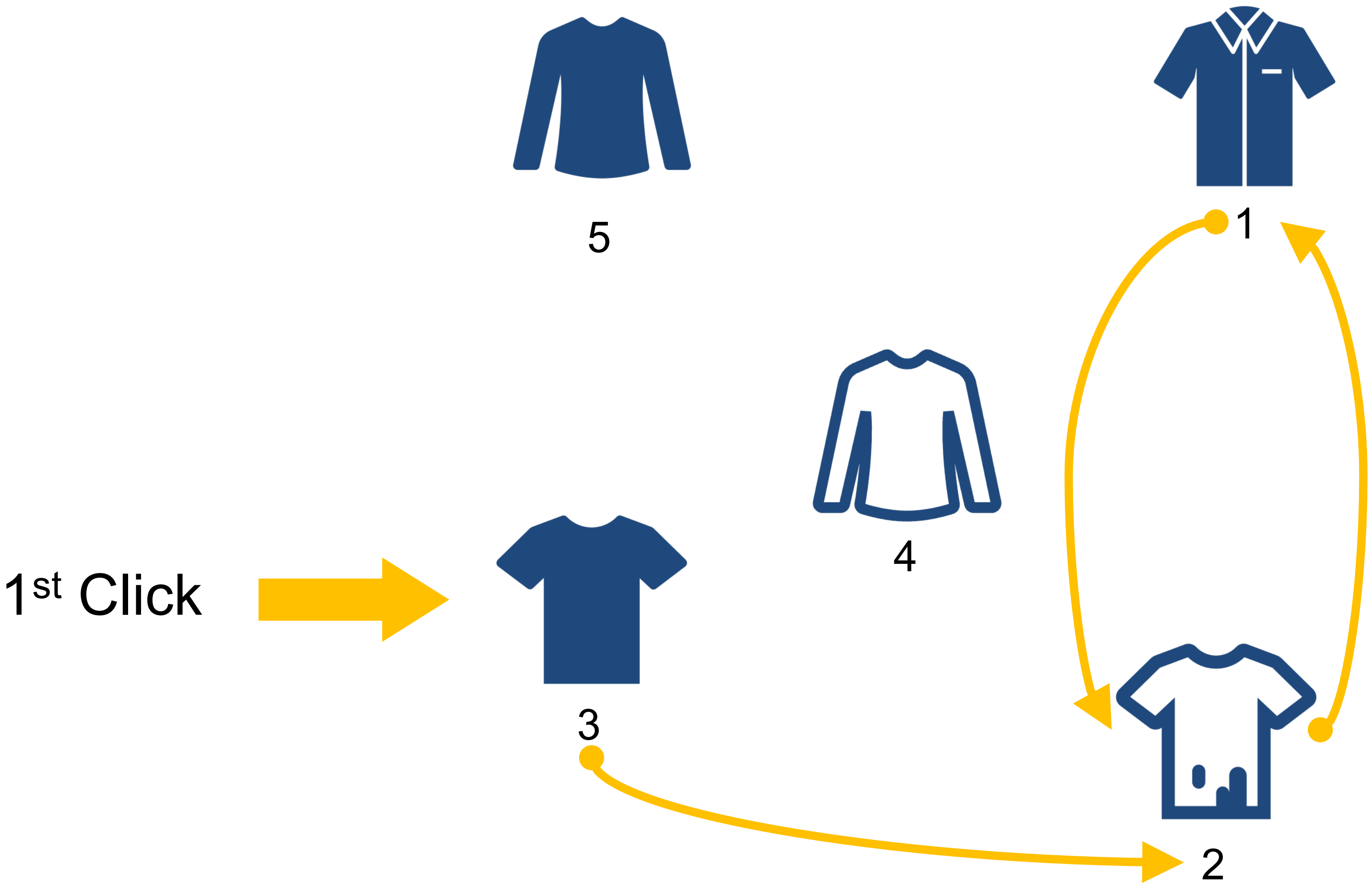

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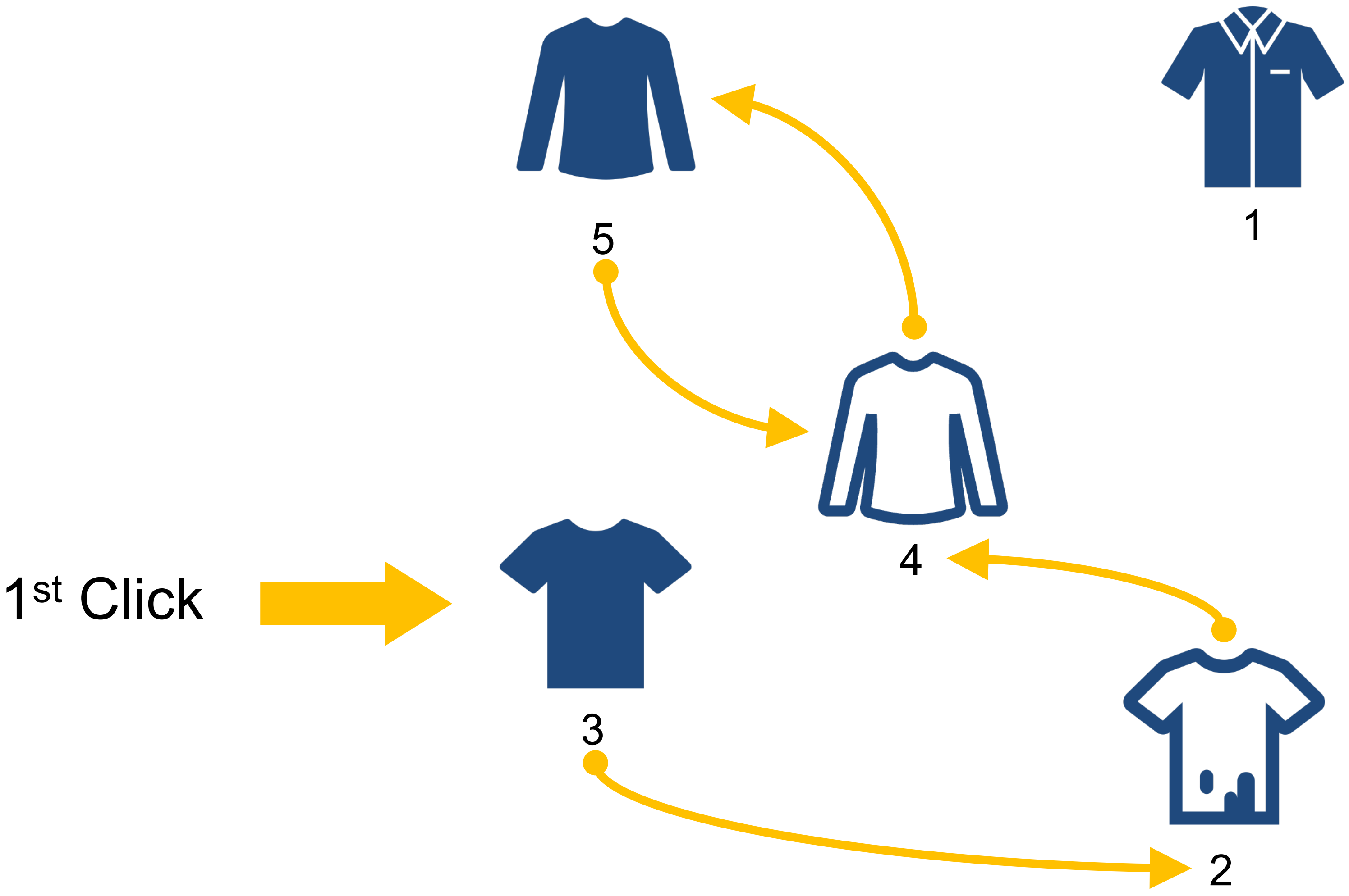

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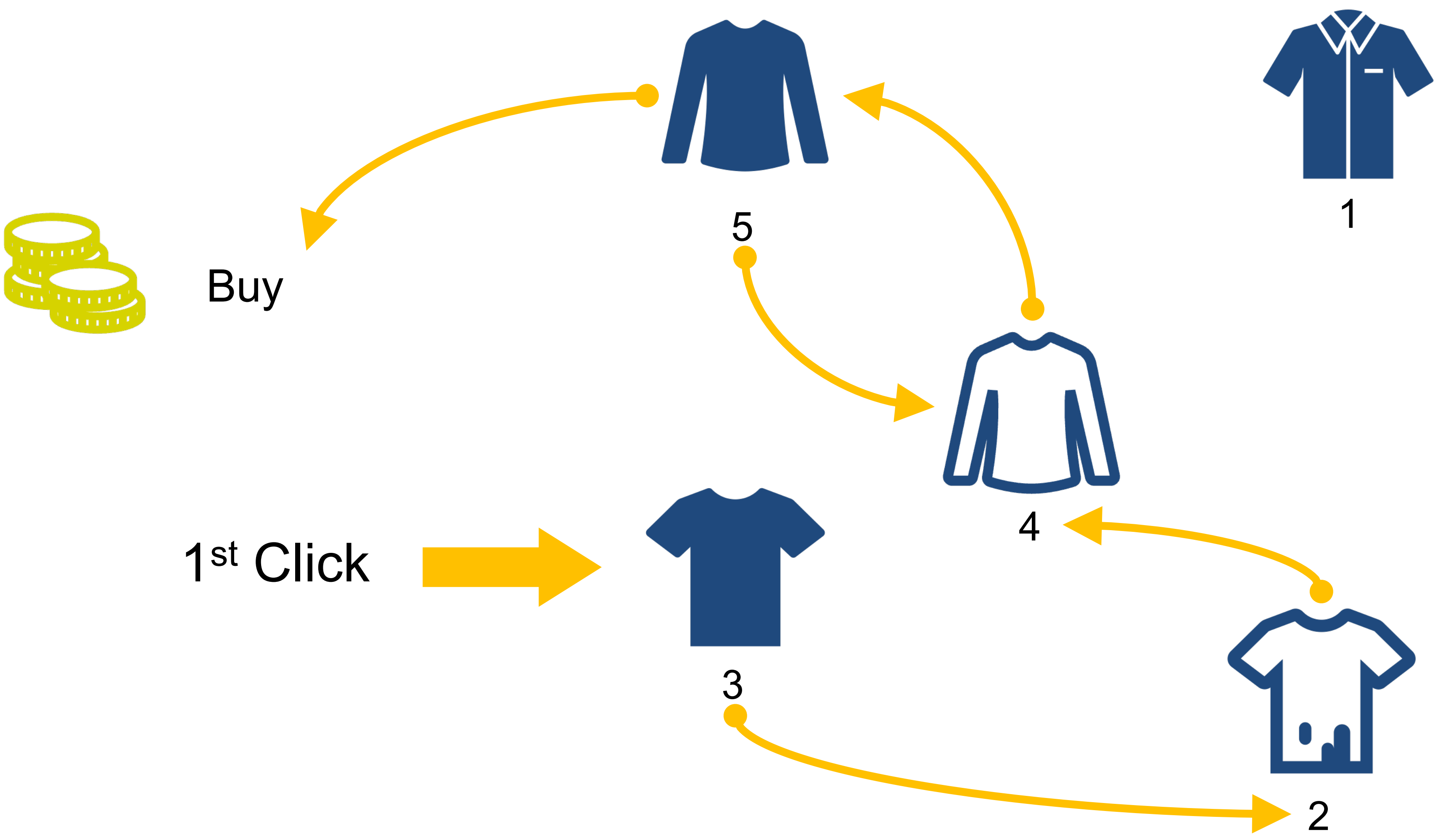
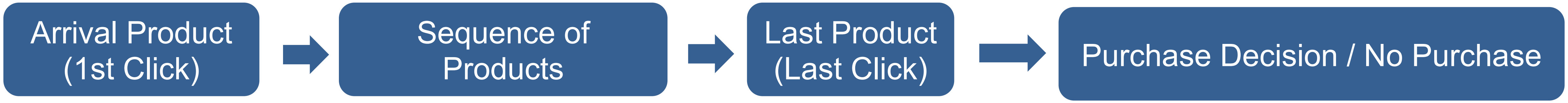


# Motivation: Click trajectories




- ➢ Click trajectories reveal the search and comparing behaviors of customers
- ➢ Click trajectories have more information than click-through rate on a single product

- Click trajectories are random and contain potential back-and-forth transitions

# Motivation: Click trajectories

- How to use click behaviors to learn the preference of customers?
  - How to model random click trajectories among millions of products?
  - How to consider the effect of assortment on the click trajectories?

- How to use click behaviors to determine the optimal prices?

# Agenda

- **Generalized Markov Chain Choice Model (GMCCM)**
- Assortment-Dependent Click Model
- Estimation
- Optimal Offline Pricing
- Dynamic Online Pricing
- Numerical Experiments

# Generalized Markov Chain Choice Model (GMCCM)

GMCCM is a choice model, independent of click behaviors

- Proposed in [Goutam et al., 2019], and [Dong et al., 2019]
- State $i$: product $i$
- State $0$: no-purchase state

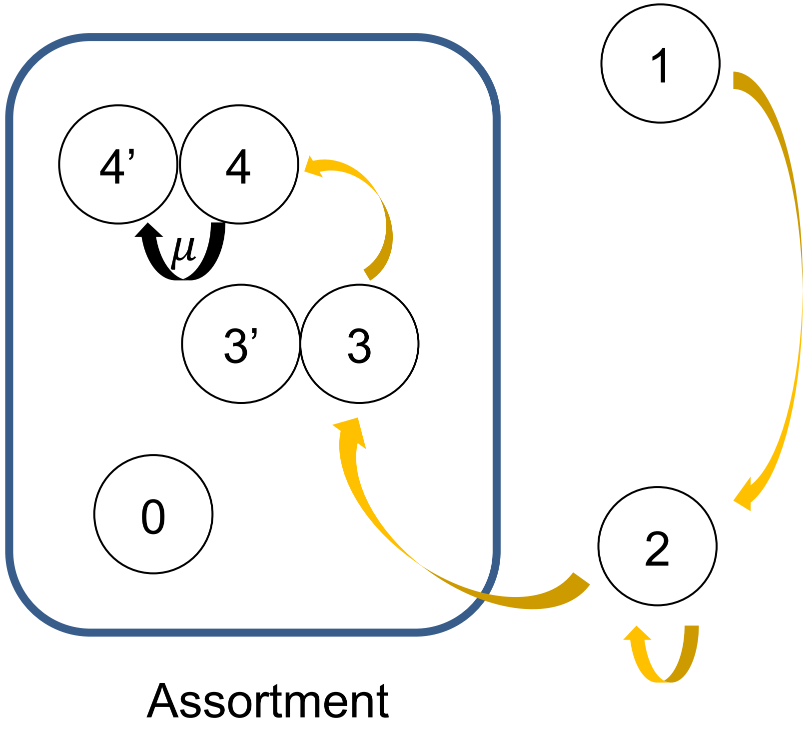
State Transition:

- If the current state is outside the assortment, keep transitioning.
- If the current state is within the assortment:

  Purchase it and leave the system with probability $\mu$;
  Otherwise, keep transitioning

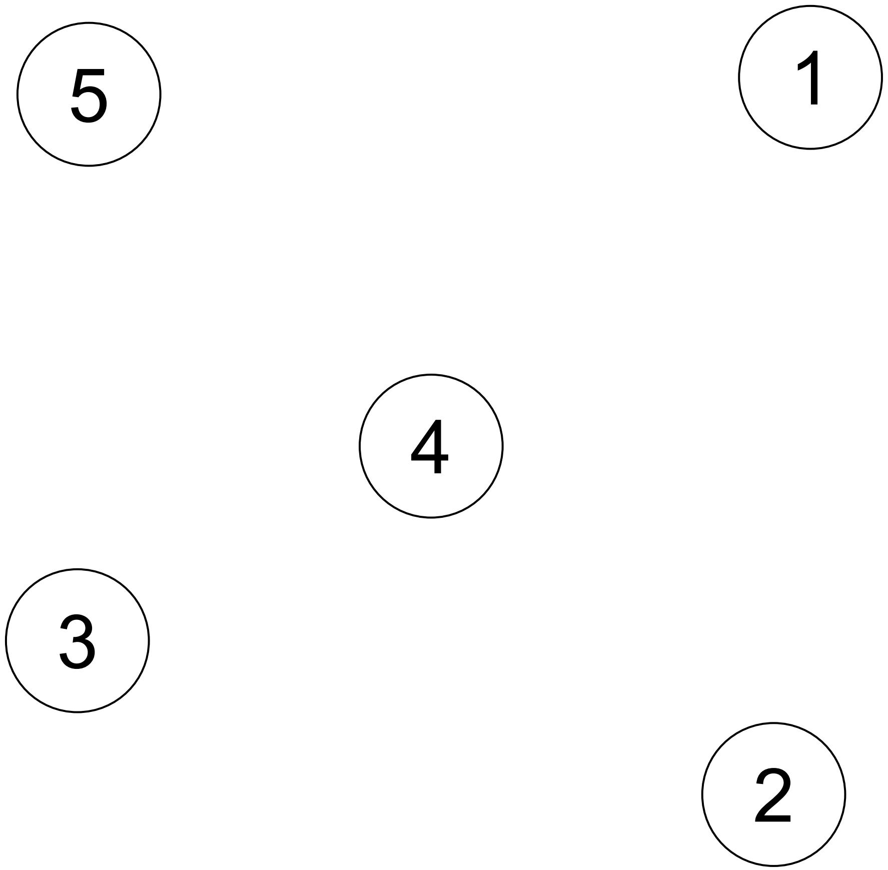
Three types of parameters:

- Transition matrix $\rho$
- Arrival probability
- Instant purchase probability $\mu$, which is a function of price, assortment, and product



# How to connect click behaviors with GMCCM?

# Each state in Markov Chain represents one product



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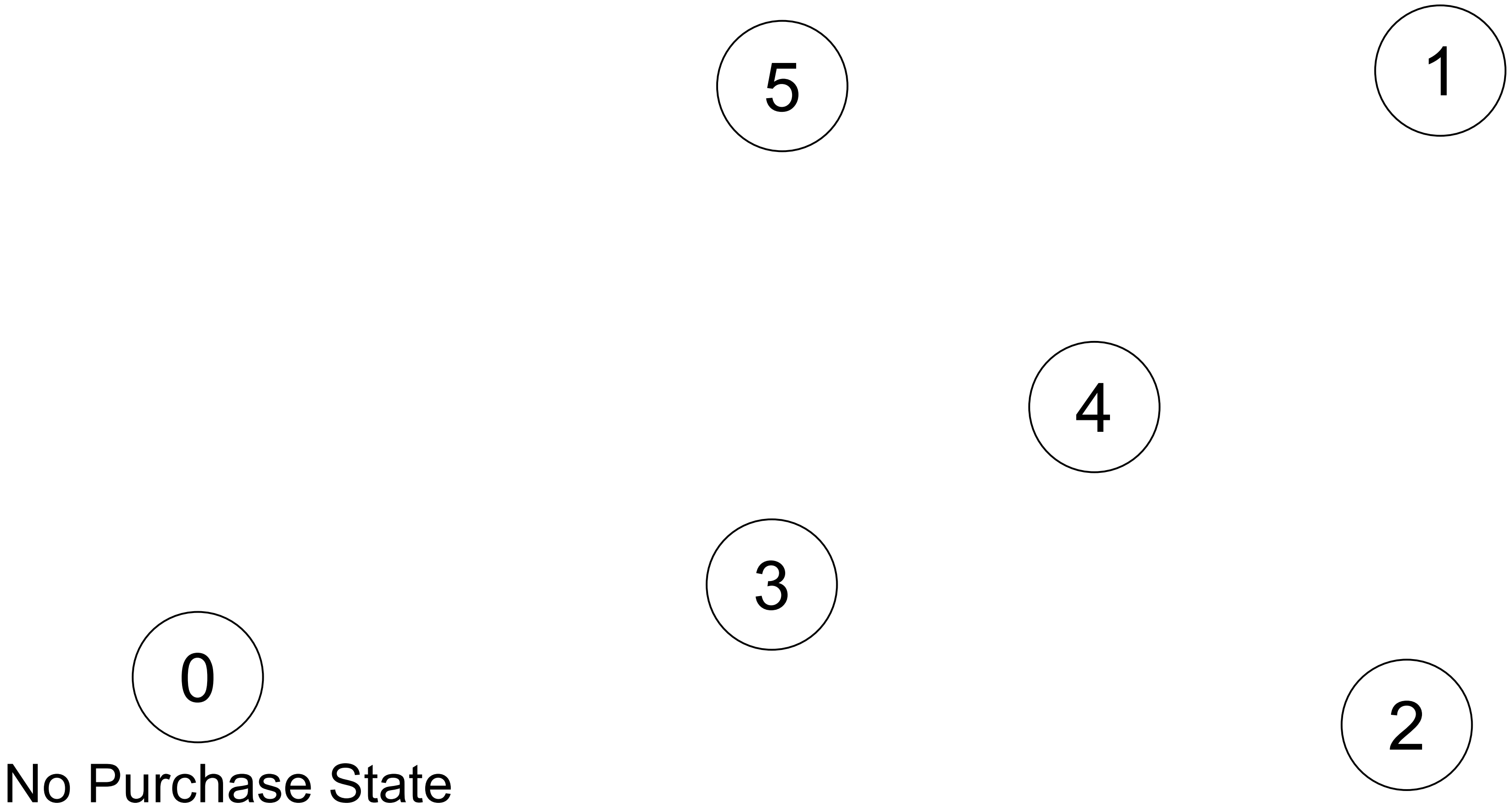
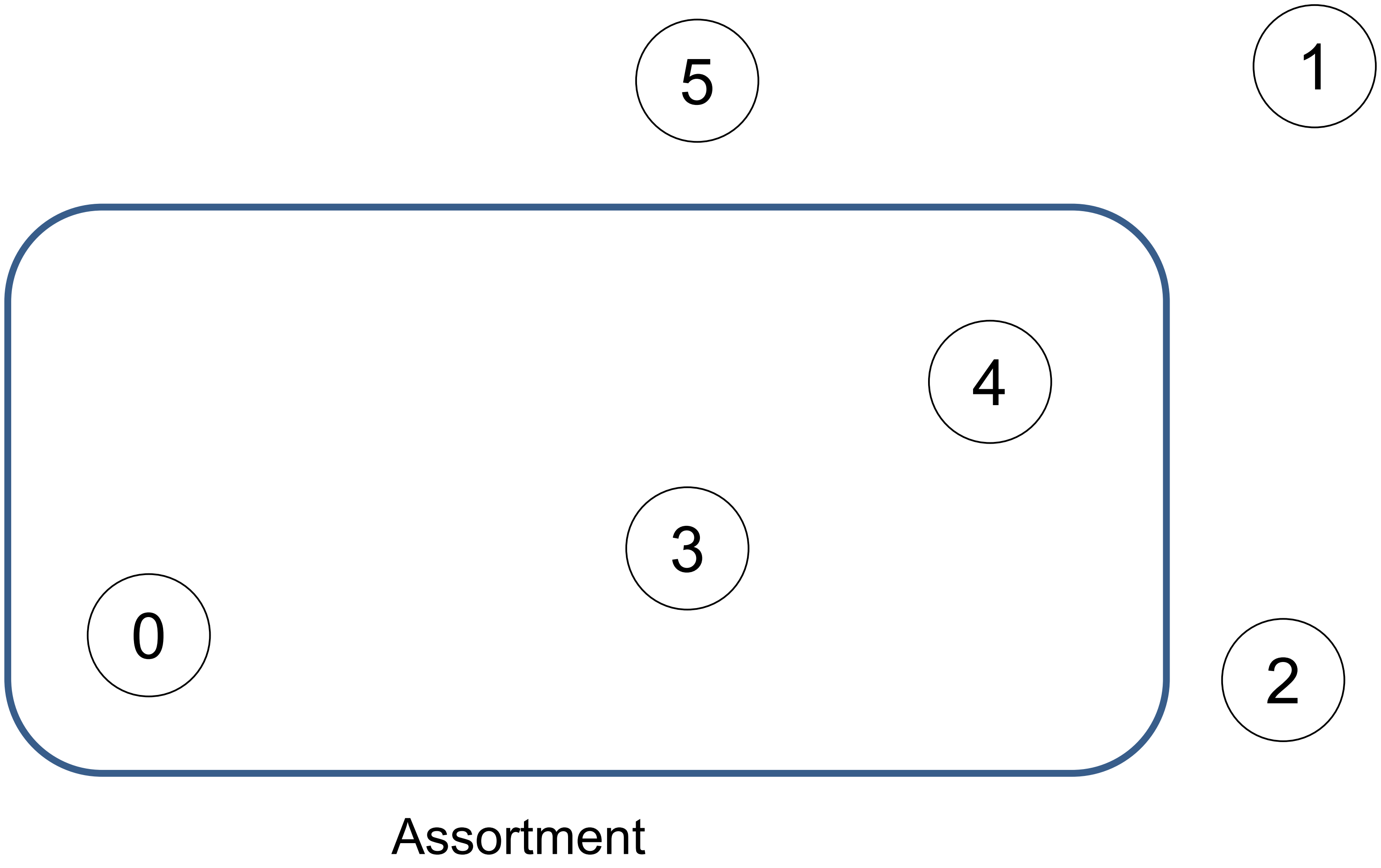


# Only products within the assortment can be clicked or purchased



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- Generalized Markov Chain Choice Model (GMCCM)

➢ **Assortment-Dependent Click Model**

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# Assortment-dependent click transition behaviors

- $\Theta_{ij}^{S}$: Click transition matrix given the assortment $S$
- $\rho_{ij}$: State transition matrix in GMCCM
- Assumption of click model:

$$\Theta_{ij}^{S} = \frac{\rho_{ij}}{\sum_{k \in \bar{S}} \rho_{ik}}, \forall i, j \in S$$

Justification 1:

- $\Theta_{ij}^{S}$ can be reduced to the choice probability in an MNL choice model
  - Utility for product $i$: $u_i$
  - By Blanchet et al. (2016), $\rho_{ij} = \frac{u_j}{1-u_i}$
  - ✓ $\Theta_{ij}^{S} = \frac{u_j}{\sum_{k \in \bar{S}} u_k}$ which follows MNL choice model

# Assortment-dependent click transition behaviors

Justification 2:

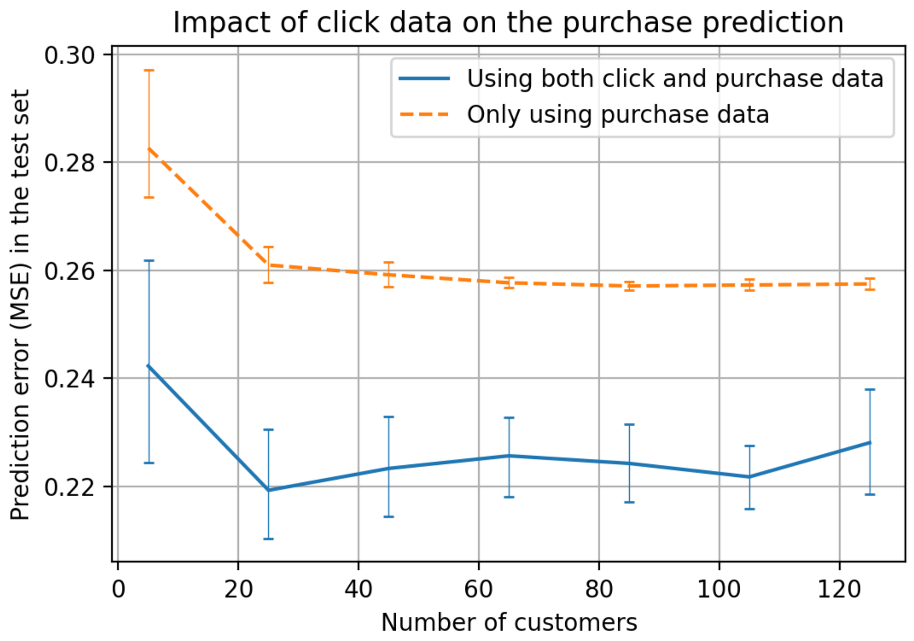
- $\Theta_{ij}^{S}$ can be viewed as the conditional probability of choosing $j$ from $\bar{S}$
  - Suppose clicking on product $i$ ⟺ visiting state $i$
  - $\Theta_{ij}^{S} = \frac{\rho_{ij}}{\sum_{k \in \bar{S}} \rho_{ik}}$ is the transition probability from $i$ to $j$, conditional on that customers only click products within $\bar{S}$

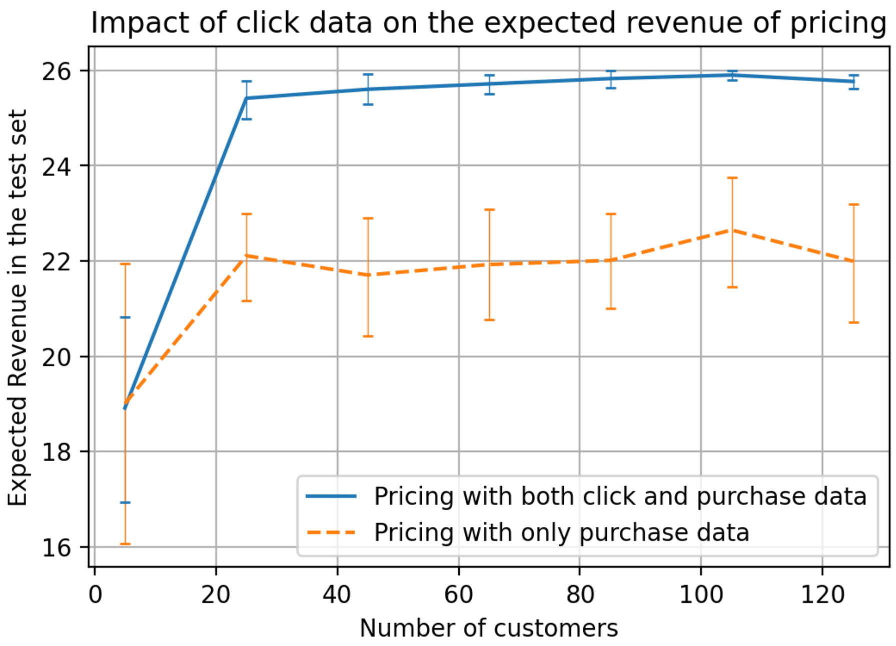
- Instant purchase probability for products within the assortment

$$\mu(i; S, \boldsymbol{p}) = e^{-\alpha_i p_i}, \forall i \in S$$

# Properties of click model

- We are the first to model the click transition behavior using a Markov chain choice model
- Click models enable us to estimate the parameters in GMCCM by click data
- Compared to the estimation methods only using purchase data
    - ✓ Using additional click data can have smaller prediction errors and better pricing decisions



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# Estimation using click model

- Scalability issue:
  - Number of products: $n$
  - Dimension of the transition matrix $\rho$: $n \times n$

- How to estimate the transition matrix efficiently?

- Consider the similarities between products
  - Products have some similar properties
  - Similar products may have similar transition patterns

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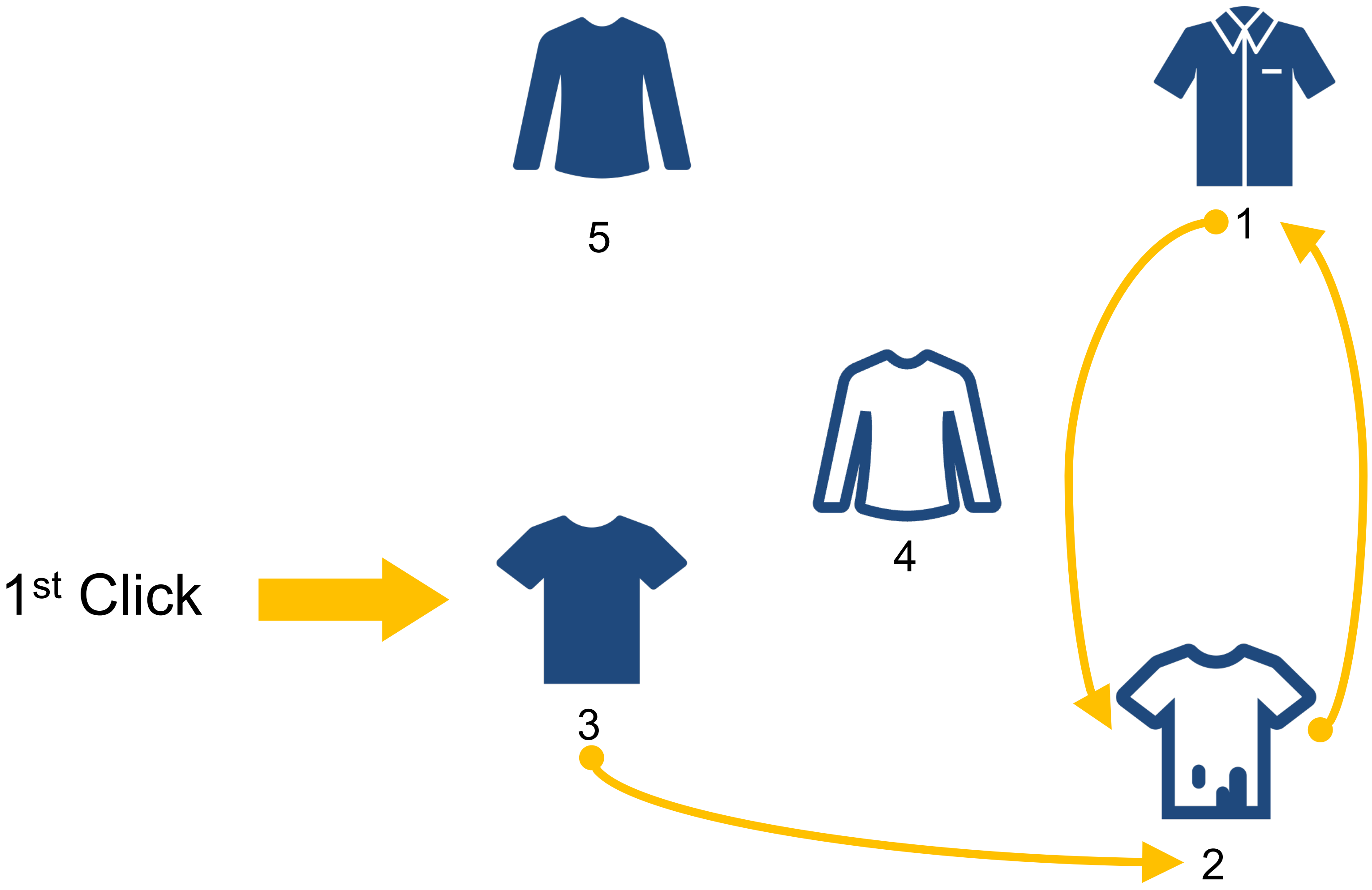

# Low-rank structure of the transition matrix

✓ Solution: Assume the rank of the transition matrix is at most $r \ll n$

- Intuition: There are potential $r$ different transition patterns among products

- Benefit of low-rank structure:
  - Reduce the search space for the transition matrix
  - Accelerate the learning rate

Under the low-rank structure, how to estimate the transition matrix efficiently?

# Estimation of the transition matrix

Minimize: negative log-likelihood of click behaviors + $\gamma||\rho||_*$

- $\gamma$: Multiplier
- $||\rho||_*$: nuclear norm of the transition matrix

- Subject to:

$$\sum_{k\in[n]} \rho_{ik} = 1 \qquad \forall i$$

$$\rho_{ij} \geq 0 \qquad \forall i,j$$

$$\rho_{00} = 1, \rho_{0j} = 0 \qquad \forall j \neq 0$$

✓ The estimation problem is restricted convex in the feasible region

- Use subgradient projection method to estimate the transition matrix

# Estimation error bound

Given a fixed assortment $S$, suppose we collect $N_S$ pairs of click transition

***Theorem***

Under certain assumptions, for any parameter $\tau \geq 1$, setting $\gamma = \frac{1}{2}\sqrt{\frac{8\tau \ln(2|S|)}{N_S \beta_1}}$, with probability at least $1 - 4(2|S|)^{-\tau/c_1}$, our estimated transition matrix $\widehat{\Theta}$ satisfies:

$$\left\|\widehat{\Theta} - \Theta^*\right\|_F \leq \frac{128}{\beta_1^2}\sqrt{\frac{2\tau r \ln(2|S|)}{N_S \beta_1}}$$

Estimation error bound:

- $\tilde{O}\left(\sqrt{r \ln(|S|)}\right)$
- Previous results in [Kallus and Udell 2020] are $\tilde{O}\left(\sqrt{r|S| \ln(|S|)}\right)$

✓ Extensions to click data from various assortments

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# Optimal pricing under GMCCM

Insights:

- For a given product $i$:

Higher click rate → More preferrable ✗→ Higher price

- The change of optimal prices depends on our defined optimal stationary revenue
  - The product's optimal stationary revenue gets higher, then the optimal prices go up
  - The product's optimal stationary revenue gets lower, then the optimal prices go down

# Optimal pricing under GMCCM

- We provide iterative algorithms that converge to the true optimal prices

**Algorithm**

At each iteration:

- For each product within the assortment:
  - Fixed the prices of other products, optimize the price of this product to maximize the revenue

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# Dynamic online pricing problem

- Given an assortment, customers arrive sequentially
- Simultaneously:
  - Estimate the parameters in GMCCM using the click data
  - Update optimal prices to maximize the revenue

Theoretical Observation:

  - Random click transition behaviors can automatically explore products

## Dynamic online pricing problem

We design an exploration-free online learning algorithm:

**Algorithm**

For $t = 1, \ldots, T$:

Customer $t$ arrives

Collect click data and purchase behavior of this customer

Add these data into the training set

Update the estimation of GMCCM model

Optimize prices based on the current GMCCM model

# Regret bound for the exploration-free online learning algorithm

- Given a fixed assortment $S$, after $T$ iterations

$$\text{Regret} \coloneqq T \max_{\boldsymbol{p}}\{\mathcal{R}(\boldsymbol{p})\} - \sum_{t=1}^{T} \mathcal{R}(\boldsymbol{p}_t)$$

**Theorem**

Under certain assumptions, Suppose the rank of the transition matrix is $r$. There exists a constant $C$ such that when $T \geq C\left(1+\ln\left(\frac{1}{\delta}\right)\right)$, with probability $1-3\delta$, the regret is at most:

$$\left(\frac{2L_2}{\underline{p}} + \frac{c_2 L_1}{\beta_1^{2.5}}\sqrt{nr}\right)\sqrt{T\ln\left(\frac{nT}{\delta}\right)}$$

- Order: $\tilde{O}(\sqrt{rnT})$ $\leq$ Regret bound without low-rank structure: $\tilde{O}(n\sqrt{T})$

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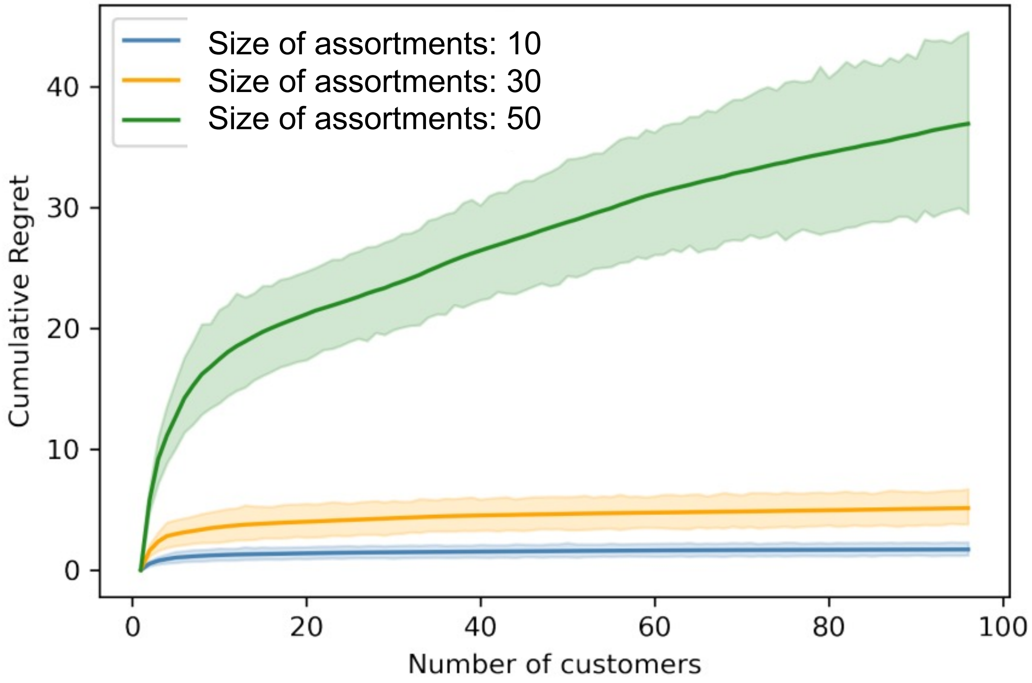
# Numerical experiments for online pricing

- Simulation based on the real-world click data
- Total products: 50



# Thank you

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